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The Ideal Form of the Skew Polynomial Ring Over Quaternion

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ABSTRACT

This research was carried out in order to develop theory about the skew polynomial ring over non-commutative ring. This study aimed to find the ideal form of the skew polynomial ring over the quaternion. The research method was the library study. In order to find the ideal form of the skew polynomial ring over the quaternion, the first thing to do was finding the endomorphism form in the quaternion ring, which was symbolized by σ, and eleven endomorphisms σ were obtained. The research results every ring had two ideals form. In general, the rings which had the identically ideal forms were categorized into three: three rings were identical, two rings were also identical, and the rest six rings had identical forms too.

Keywords: commutative ring, endomorphism, non-commutative ring, quaternion, skew polynomial ring.

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INTRODUCTION

Ring is all about one set with two operations, addition (+) and multiplication (x). If the multiplication of the every two elements of the ring is commutative, it is called commutative ring. If the multiplication of a ring's element is not commutative, it is called non-commutative ring.

Ore [8] introduced the skew polynomial ring, the development of ring. The skew polynomial ring contains a set of skew polynomials with non-commutative multiplication. For example, the skew polynomial ring over real numbers is the set of polynomials $a_n x^n + a_{n-1}x^{n-1} + \dots + a_2 x^2 + ax + a_0$ where x is an unknown variable and $a_i \in \mathbb{R}$ with the multiplication rule $xa = \sigma(a)x + \delta(a)$ for each $a_i \in \mathbb{R}$ while σ is an endomorphism and δ is σ-derivative.

Amir [3], it says that the researchers of skew polynomial ring can be divided into three groups. The first group develops the class of skew polynomial ring to be a bigger class of ring. The second group uses skew polynomial ring to be more applicative, while the third group observes the structure of the skew polynomial ring by using various base rings. In the dissertation, Amir is in the third group with the base ring in Dedekind domain.

Amir [1] has discussed the characteristics of the σ-prime ideal and the skew polynomial ring. Amir [2] discussed the characteristics of the ideal in the skew polynomial ring. Then, he improved his research [3] in the skew polynomial ring where the base ring is commutative ring in Dedekind domain. Amir et al [4] found the ideal form in the skew polynomial ring with the different commutative base ring and Amir [5] started to expand the skew polynomial ring in non-commutative ring, quaternion. In his research, he discovered one of endomorphism forms of the quaternion ring which later became a ring. Because of those reasons, we decided to develop Amir's research about the skew polynomial ring with non-commutative base ring, quaternion.

Quaternion was found by Hamilton. For his merit, the quaternion is denoted by $\mathbb{H}$. The purpose of this research is to find the ideal form of the skew polynomial ring which is quaternion base ring.

There are some definitions supporting this research. They are homomorphism ring, endomorphism, and ideal. Based on Rotman [9], ring is a set with two binary operations, multiplication and addition, satisfying the rule as an abelian group, multiplicatively closed, satisfying the associative and distributive property of multiplication, and having inverse for multiplication.

Fraleigh [6] defined homomorphism group as a function mapping *G* to *G'* that is $\varphi: G \rightarrow G'$ with the rule $\varphi(ab) = \varphi(a)\varphi(b), \forall\, a, b \in G$ where ab multiplication satisfies the operation in the left side of G and $\varphi(a)\varphi(b)$ satisfies the operation in the right side of *G'*. The mapped homomorphism ring is ring. For example, $\mathcal{R}$ $'$ $\forall\, a, b \in$ satisfies $\varphi(a + b) = \varphi(a) + \varphi(b)$ and $\varphi(ab) = \varphi(a)\varphi(b)$ $\varphi: \rightarrow$, mapping is homomorphism ring.

Rotman [9] stated that a homomorphism group φ is said to be endomorphism if φ is a function associating from G to itself whereas a homomorphism ring is said to be endomorphism ring if it relates ring $\mathcal{R}$ to itself.

The definition of ideal I by Fraleigh [6] is a sub-ring of ring $\mathcal{R}$ if $\forall r \in I$ and

$ra \in I, \forall a \in I, r \in$ □ . If I is an ideal in $\mathcal{R}$, the multiplication between the elements of I and the elements of $\mathcal{R}$ must obtain an element in I.

According to McConnel and Robson [7], □ is a ring with identity 1, σ is an endomorphism of , and δ is a σ-derivative which is an endomorphism of , as a multiplication group, and $\delta(ab) = (a)\delta(b) + \delta(a)b$ for each $a, b \in$ □. The skew polynomial ring over with variable x is a ring, $[x;\sigma;] = \{f(x) = a_n x^n + \cdots + a_0 \mid a_i$ } with $xa = \sigma(a)x + \delta(a); \forall a \in$ □. An element p of the skew polynomial ring $[x;]$ has the canonical form $p = \sum_{i=0}^{r} a_i x^i$, $r \in$ + = {0, 1, …}, $a_i \in$ □, $i = 1, 2, \ldots, r$.

Based on Shomake (2007), quaternion is a linear combination with real scalars and three units of orthogonal imaginary, denoted by ***i, j*** and ***k***, with real coefficients written as □ = $\{q = q_0 + \boldsymbol{i}q_1 + \boldsymbol{j}q_2 + \boldsymbol{k}q_3 \mid q_0, q_1, q_2, q_3 \in$ □ $\}$ where $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ rule $\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = -1$; $\boldsymbol{ij} = -\boldsymbol{ji} = \boldsymbol{k}$; $\boldsymbol{jk} = -\boldsymbol{kj} = \boldsymbol{i}$; $\boldsymbol{ki} = -\boldsymbol{ik} = \boldsymbol{j}$. For example, $a = a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k} = [a_0, \boldsymbol{a}]$ and $b = b_0 + b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k} = [b_0, \boldsymbol{b}]$ with $\boldsymbol{a}, \boldsymbol{b} \in$ □^{3}. The addition and $a + b = [a_0 + b_0, \boldsymbol{a} + \boldsymbol{b}]$, 2) $ab = [a_0 b_0 - \boldsymbol{a} \cdot \boldsymbol{b}, a_0\boldsymbol{b} + b_0\boldsymbol{a} + \boldsymbol{a} \times \boldsymbol{b}]$.

The purpose of this research is to discover the ideal form of the skew polynomial ring over the quaternion ring.

DATA AND METHOD

Research location

This research takes place in Mathematics Department, FMIPA, Hasanuddin University. This is a theoretical research. It is done by finding the form of endomorphism of quaternion ring then every endomorphism form will become the particular ring and we can obtain the ideal form from the ring.

RESULT

The skew polynomial ring that will be arranged is a skew polynomial ring and the base ring is a quaternion denoted as $\mathbb{H}$. Thus, the first step is to form the endomorphism in quaternion $\mathbb{H}$.

Theorem 1

For example, $a = a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k} \in$ □ and $\sigma :$ □ $\to$ □, we can yield some endomorphism forms in *quaternion* $\mathbb{H}$

$$\sigma_1(a) = \sigma_1(a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}) = (a_0 - a_1\boldsymbol{i} - a_2\boldsymbol{j} + a_3\boldsymbol{k})$$

$$\sigma_2(a) = \sigma_2(a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}) = (a_0 - a_1\boldsymbol{i} + a_2\boldsymbol{j} - a_3\boldsymbol{k})$$

$$\sigma_3(a) = \sigma_3(a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}) = (a_0 + a_1\boldsymbol{i} - a_2\boldsymbol{j} - a_3\boldsymbol{k})$$

$\sigma_4\ a \quad \sigma_4\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$
$a_0 \quad a_2\boldsymbol{i} \quad a_3\boldsymbol{j} \quad a_1\boldsymbol{k}$
$\sigma_5\ a \quad \sigma_5\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$
$a_0 \quad a_2\boldsymbol{i} \quad a_3\boldsymbol{j} \quad a_1\boldsymbol{k}$
$\sigma_6\ a \quad \sigma_6\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$
$a_0 \quad a_2\boldsymbol{i} \quad a_3\boldsymbol{j} \quad a_1\boldsymbol{k}$
$\sigma_7\ a \quad \sigma_7\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$
$a_0 \quad a_2\boldsymbol{i} \quad a_3\boldsymbol{j} \quad a_1\boldsymbol{k}$
$\sigma_8\ a \quad \sigma_8\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$
$a_0 \quad a_3\boldsymbol{i} \quad a_1\boldsymbol{j} \quad a_2\boldsymbol{k}$
$\sigma_9\ a \quad \sigma_9\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$
$a_0 \quad a_3\boldsymbol{i} \quad a_1\boldsymbol{j} \quad a_2\boldsymbol{k}$
$\sigma_{10}\ a \quad \sigma_{10}\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$
$a_0 \quad a_3\boldsymbol{i} \quad a_1\boldsymbol{j} \quad a_2\boldsymbol{k}$
$\sigma_{11}\ a \quad \sigma_{11}\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$
$a_0 \quad a_3\boldsymbol{i} \quad a_1\boldsymbol{j} \quad a_2\boldsymbol{k}$

$\sigma_1\ a_0$
$a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k} \quad a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j}$
$a_3\boldsymbol{k}$

$a \quad a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$
$b \quad b_0 \quad b_1\boldsymbol{i} \quad b_2\boldsymbol{j} \quad b_3\boldsymbol{k}$
$\sigma\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$
$a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$

$\sigma\ a \quad b \quad \sigma\ a \quad \sigma\ b$
$\sigma\ ab \quad \sigma\ a \quad \sigma\ b$
$\sigma\ a \quad b \quad \sigma\ a \quad \sigma\ b$

$\sigma\ a \quad b \quad \sigma\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$
$b_0 \quad b_1\boldsymbol{i} \quad b_2\boldsymbol{j} \quad b_3\boldsymbol{k}$
$\sigma\ a_0 \quad b_0 \quad a_1 \quad b_1\ \boldsymbol{i} \quad a_2$
$b_2\ \boldsymbol{j} \quad a_3 \quad b_3\ \boldsymbol{k}$
$a_0 \quad b_0 \quad a_1 \quad b_1\ \boldsymbol{i}$
$a_2 \quad b_2\ \boldsymbol{j} \quad a_3$
$b_3\ \boldsymbol{k}$
$\sigma\ a \quad \sigma\ b \quad \sigma\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j}$
$a_3\boldsymbol{k} \quad \sigma\ b_0 \quad b_1\boldsymbol{i}$
$b_2\boldsymbol{j} \quad b_3\boldsymbol{k}$
$a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k} \quad b_0$
$b_1\boldsymbol{i} \quad b_2\boldsymbol{j} \quad b_3\boldsymbol{k}$
$a_0 \quad b_0 \quad a_1 \quad b_1\ \boldsymbol{i} \quad a_2$
$b_2\ \boldsymbol{j} \quad a_3 \quad b_3\ \boldsymbol{k}$
$\sigma\ a \quad b \quad \sigma\ a \quad \sigma\ b$
$\sigma\ ab \quad \sigma\ a \quad \sigma\ b$
$\sigma\ ab \quad \sigma\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k} \quad b_0$
$b_1\boldsymbol{i} \quad b_2\boldsymbol{j} \quad b_3\boldsymbol{k}$
$\sigma\ a_0b_0 \quad a_1b_1 \quad a_2b_2 \quad a_3b_3$
$a_0b_1 \quad a_1b_0 \quad a_2b_3$
$a_3b_2\ \boldsymbol{i}$
$a_0b_2 \quad a_1b_3 \quad a_2b_0$
$a_3b_1\ \boldsymbol{j}$
$a_0b_3 \quad a_1b_2 \quad a_2b_1$
$a_3b_0\ \boldsymbol{k}$
$a_0b_0 \quad a_1b_1 \quad a_2b_2 \quad a_3b_3$
$a_0b_1 \quad a_1b_0 \quad a_2b_3$
a_3b_2
$a_0b_2 \quad a_1b_3$
$a_2b_0 \quad a_3b_1\ \boldsymbol{j}$
$a_0b_3 \quad a_1b_2 \quad a_2b_1$
$a_3b_0\ \boldsymbol{k}$

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$\sigma(a) = \sigma(a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k})$

$\sigma(b) = \sigma(b_0 + b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k})$

$(\;) = (a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k})$

$= (b_0 - b_1\boldsymbol{i})$

$(\;) = (+ + b_2\boldsymbol{j} + b_3\boldsymbol{k})$

$a_0b_0 - (a_1b_1 + a_2b_2 + a_3b_3$

$a_0b_1 \quad a_1b_0 \;)$

$= (+ a_2b_3 \quad a_3b_2 \;)\boldsymbol{i}$

$(\;) = (+ + a_0b_2 \quad a_1b_3$

$= (+ a_2b_0 \quad a_3b_1 \;)\boldsymbol{j}$

$a_0b_3 \quad a_1b_2$

$= (- a_2b_1 \quad a_3b_0 \;)\boldsymbol{k}$

$(\;) = (\sigma(ab) + \sigma(a) + \sigma(b))$

$(\;) = (+ + +)$

$(\;) = (+ + +)$

$= (+ - -)$

Proof

We want to prove that $(+ +) = (x\,\sigma_1 \quad x\,\sigma_2 + x\,\sigma_3)$ is an endomorphism ring quaternion. $\sigma_1, \sigma_2, \sigma_3$ $a = a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$

$+ \; \sigma_1(a) = \sigma_1(a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}) \quad a_0 = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k} +$

$\sigma_2(a) = \sigma_2(a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}) = a_0 \quad a_1\boldsymbol{i} + a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$

For example, there are two quaternions endomorphism ring quaternion, then it $\sigma_3(a) = \sigma_3(a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}) = a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k}$

If $(x\,\sigma_1) = x\,(\sigma_2) \cdot (\;).\; x\,\sigma_3$ is an $I \quad x^2 \quad x\,\sigma_1$

$I \quad x^2 \quad x\,\sigma_1$

$(+) = [(+ + +)$

$I \quad x^2 \quad x\,\sigma_2 + (+ + +)]$

$I \quad x^2 \quad x\,\sigma_2$

$I \quad x^2 \quad (x\,\sigma_3 +) + (\;) + (\;) + (\;)$

$I \quad x^2 = (+ x\,)\sigma_3 - (+) \quad)$

$- (+)$

$+ ($

$(Iq \quad x + I$

$q \quad x \qquad x\,\sigma_1$

$+ \quad)$

$q \quad x \quad q_0 \quad q_1x \quad q_2x^2 \quad q_3x^3$

$q_4x^4 \quad q_tx^t \quad \sum_{n=0}^{t} q_nx^n$

$= (Iq\,\bar{x} \quad x^2 \quad x\,\sigma_1 \left[\sum_{n=0}^{t} q_nx^n\right]$

$(x^2 = \; _0 \; (\;)_1 x \cdot (\;)_2 x^2 \quad _3 x^3$

$= (+) - (+) - ($

$(\;) = [(\; _s x^s \left[\sum_{n=0}^{t} q_nx^n\right]$

$= [((+) = (\;) + (\;) (\;)$

$x^2 \left[\sum_{n=0}^{s} \; _n x^n\right]\left[\sum_{n=0}^{t} q_nx^n\right]$

$x^2 \left[\sum_{u+v=0} \; _u(\sigma_1{}^u \; q_v)\,x^0\right.$

$\sum_{u+v=1} \; _u(\sigma_1{}^u \; q_v)\,x^1 \; -$

$= (\sum_{u+v=2} \; _u \; \sigma_1{}^u \; q_v)\,x^2$

$\sum_{u+v=3} \; _u \; \sigma_1{}^u(q_v)\,x^3$

$- (-$

$\left.\sum_{u+v=s+t} \; _u \; \sigma_1{}^u \; q_v \; + x^{s+t}\right]$

satisfies

$+\ ) =\ (\ ) +\ (\ )$

$(\ +\ ) =\ (\ ) +\ (\ )$

$x^2 \sum_{u+v=0} {}_u(\sigma_1{}^u\ q_v\ )\, x^0$

$x^2 \sum_{u+v=1} {}_u(\sigma_1{}^u\ q_v\ )\, x^1$

$x^2 \sum_{u+v=2} {}_u\ \sigma_1{}^u\ q_v\quad x^2$

$x^2 \sum_{u+v=3} {}_u\ \sigma_1{}^u\ q_v\quad x^3$

$x^2 \sum_{u+v=s+t} {}_u\ \sigma_1{}^u\ q_v\quad x^{s+t}$

$\sigma_1{}^2\left(\sum_{u+v=0} {}_u(\sigma_1{}^u\ q_v\ )\right)x^2$

$\sigma_1{}^2\left(\sum_{u+v=1} {}_u(\sigma_1{}^u\ q_v\ )\right)x^3$

$\sigma_1{}^2\left(\sum_{u+v=2} {}_u(\sigma_1{}^u\ q_v\ )\right)x^4$

$\sigma_1{}^2\left(\sum_{u+v=3} {}_u(\sigma_1{}^u\ q_v\ )\right)x^5$

$\sigma_1{}^2\left(\sum_{u+v=s+t} {}_u(\sigma_1{}^u\ q_v\ )\right)x^{s+t+2}$

$\sigma_1{}^2\ a\qquad a$

$Iq\ \ x$

$\sum_{u+v=0} {}_u(\sigma_1{}^u\ q_v\ )\, x^2$

$\sum_{u+v=1} {}_u(\sigma_1{}^u\ q_v\ )\, x^3$

$\sum_{u+v=2} {}_u\ \sigma_1{}^u\ q_v\quad x^4$

$\sum_{u+v=3} {}_u\ \sigma_1{}^u\ q_v\quad x^5$

$\sum_{u+v=s+t} {}_u\ \sigma_1{}^u\ q_v\quad x^{s+t+2}$

$+$

$+\ ($

$Iq\ \ x$

$+$

$\left[\sum_{n=0}^{s+t}\sum_{u+v=n} {}_u(\sigma_1{}^u\ q_v\ )x^n\right]x^2$ 72

$x^2 q\ \ x\qquad x^2\left[\sum_{n=0}^{t} q_n x^n\right]$

$\sigma_1{}^2\ q_0\ \ x^2\qquad \sigma_1{}^2\ q_1\ \ x^3$

$\sigma_1{}^2\ q_2\ \ x^4\qquad\qquad \sigma_1{}^2\ q_t\ \ x^{t+2}$

$q_0 x^2\quad q_1 x^3\quad q_2 x^4$

$q_0 x^{t+2}$

$x^2 q\ \ x\qquad \left[\sum_{n=0}^{t} q_n x^n\right]x^2$

$q\ \ x\ \ x^2$

$Iq\ \ x$

$x^2\left[\sum_{n=0}^{s+t}\sum_{u+v=n} {}_u(\sigma_1{}^u\ q_v\ )x^n\right]$

$x^2\quad x\ \ \sigma_1$

$Iq\ \ x\qquad I$

$q\ \ x\ \ I\qquad I$

$q\ \ x\qquad\quad x\ \ \sigma_1$

$q\ \ x\qquad q_0\quad q_1 x\quad q_2 x^2\quad q_3 x^3$

$q_4 x^4\qquad\qquad q_t x^t$

${}_{n=0}^{t}\, q_n x^n\ \ q_n$

$q\ \ x\ \ I$

$\left[\sum_{n=0}^{t} q_n x^n\right]x^2\quad x\ \ \sigma_1$

$)$

$+\qquad -$

$)$

$$\left[\sum_{n=0}^{t} q_n x^n\right]$$

$$\left[\sum_{n=0}^{t} q_n x^n\right]\left[\sum_{n=0}^{s} x^n\right] x^2$$

It is proved that

$$\sum_{u+v=0} q_u(\sigma_1^{u}\ \ v)\,x^0$$

$$\sum_{u+v=1} q_u(\sigma_1^{u}\ \ v)\,x^1$$

$$\sum_{u+v=2} q_u(\sigma_1^{u}\ \ v)\,x^2$$

$$\sum_{u+v=3} q_u(\sigma_1^{u}\ \ v)\,x^3$$

$$\sum_{u+v=s+t} q_u(\sigma_1^{u}\ \ v)\,x^{s+t}\,x^2$$

$$\left[\sum_{n=0}^{s+t}\sum_{u+v=n} q_u(\sigma_1^{u}\ \ v)x^n\right]x^2$$

The next point proof will not be given. It is because the proof is just the same as before.

Theorem 2

The skew polynomial ring over quaternion is given , and while are endomorphism where such that

The ideals of the skew polynomial rings

Proof

There are two steps to prove the theorem

1) We will show that ( ) ⊆ where

$a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$

Suppose that

$x\ \sigma_4 + \cdots + x\ \sigma_8$

$x\ \sigma_5 \quad x\ \sigma_6 \quad x\ \sigma_7 \quad x\ \sigma_9 \quad x\ \sigma_{10} \quad x\ \sigma_{11}$

$\sigma_5\ a \quad \sigma_5\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k} \quad a_0 \quad a_2\boldsymbol{i} \quad a_3\boldsymbol{j} \quad a_1\boldsymbol{k}$

$\sigma_6\ a \quad \sigma_6\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k} \quad a_0 \quad a_2\boldsymbol{i} \quad a_3\boldsymbol{j} \quad a_1\boldsymbol{k}$

$\sigma_7\ a \quad \sigma_7\ a_0 \quad a_1\boldsymbol{i} \quad a_2\boldsymbol{j} \quad a_3\boldsymbol{k} \quad a_0 \quad a_2\boldsymbol{i} \quad a_3\boldsymbol{j} \quad a_1\boldsymbol{k}$

, and are

= □

σ_9 a σ_9 a_0 $a_1\boldsymbol{i}$ $a_2\boldsymbol{j}$

[; $a_3\boldsymbol{k}$ a_0 $a_3\boldsymbol{i}$ $a_1\boldsymbol{j}$ $a_2\boldsymbol{k}$

σ_{10} a σ_{10} a_0 $a_1\boldsymbol{i}$ $a_2\boldsymbol{j}$

$a_3\boldsymbol{k}$ a_0 $a_3\boldsymbol{i}$ $a_1\boldsymbol{j}$ $a_2\boldsymbol{k}$

σ_{11} a σ_{11} a_0 $a_1\boldsymbol{i}$ $a_2\boldsymbol{j}$

$a_3\boldsymbol{k}$ a_0 $a_3\boldsymbol{i}$ $a_1\boldsymbol{j}$ $a_2\boldsymbol{k}$

x σ_5 x σ_6 x σ_7 x σ_9 x σ_{10}

x σ_{11}

I x^6 x σ_5

I x^6 x σ_5

I x^6 x σ_6

I x^6 x σ_6

I x^6 x σ_7

I x^6 x σ_7

I x^6 x σ_9

I x^6 x σ_9

I x^6 x σ_{10}

I x^6 x σ_{10}

I x^6 x σ_{11}

I x^6 x σ_{11}

σ

x σ

x $\sum_{i=0}^{n} q_i x^i$ q_i q_i q_0

$q_1\boldsymbol{i}$ $q_2\boldsymbol{j}$ $q_3\boldsymbol{k}$ q_0 q_1 q_2 q_3

Note that

Because

Then,

2) We want to show that $(\ ) \subseteq$,

Hence, $(\ ) \subseteq [\ ;\ ]$

where

Suppose that

$$+ \quad \square \; (\quad (\quad)) \quad + \cdots \qquad \square \, [\; ; \quad]$$

$$+ \quad \square \; (\quad (\quad))$$

= [(□ + □

+ □ + □

+ ⋯ + □)]

= [+ (□) (□)

)

+ (□

+ ⋯

+ (□)]

=

= [(□)

+ (□) +

∑ (□)

∑ (□) +

+

∑

⋯ +

(□)]

= (□)

( ) = ⊆ □ (□)

Hence, ( ) ⊆

Thus, *I* is an ideal because ( ) and [;]

are subring of *I*.

( )

Theorem 3

The skew polynomial rings over quaternion □[;] and □[;] are given and , are endomorphisms where

= + + + ∈ □ such that

( ) = (+ +

+ +

+

and

( ) = (+ + +

) = +

.

The ideal of the skew polynomial rings and are

) = + + +

= □[;]

]

□[;] □[;]

= □[;]

+ 1)□[;]

= (+ 1)□[;

Theorem 4

= (

The skew polynomial rings over quaternion

□[;], □[;], □[;], □

[;],

□[;],

= + +

+ □[;]

∈ □ and are

, , (,) , =

endomorphism where + +

(+ such that +

(+ + +

) =

) = = ∓ − ;

( ) ≡

(+ + +

+ − − ,

) =

with the same way.

The reader can prove part b, c, d, e, and f

( ) = (+ + +

) = − − + ,

( ) = (+ + +

− , and

( ) = −(+ +

− +

The ideal of the skew polynomial rings) = + − ;], and are [;], [;], [;], [;], [;]

[;]

[;]
= [;]
= [

[;]
= (+ 1)
= [;]

[;]
= (+ 1)
= [;]

[;]
= (+ 1)
= [;]

;]
= (+ 1)
= [;]

;]
= (+ 1) [

= (+ 1) [

DISCUSSION

The skew polynomial ring over quaternion is a skew polynomial ring with the quaternion and endomorphism ring, denoted by σ, as the base ring. The skew polynomial ring over quaternion [;] is completely defined as a ring formed from polynomial rings with unknown variable *x*, ( ) = ∑ ∈ = + + with + , , ,

This research provides an endomorphism form σ of quaternion ring. Each of endomorphism forms σ becomes the ring itself. Each of the skew polynomial rings has two ideal forms. Generally, there are three groups of the identic ideals of the ring. Three rings are identic ideals, two rings are the same in the shape, and the last six rings are identic ideals too.

CONCLUSION AND SUGGESTION

The research shows that there are twenty two ideal forms and every ring has two ideals. There are still many possibilities to find more ideal forms from this research. For the next research, we hope the reader can explore other ideal forms besides what are founded in this research.

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