International Journal of Engineering and Science Application
ISSN 2406-9833
IJEScA

Modified Yield-Line Theory for Prestressed Concrete Deck Slabs with Interface between Old and New Deck Slabs

Fakhruddin[1*], R. Djamaluddin[1], R. Irmawaty[1] J. Niwa[2]

[1]Department of Civil Engineering, Faculty of Engineering, Hasanuddin University
Jl. Poros Malino Km. 6 Gowa, 92172, Indonesia
[2]Department of Civil and Environmental Engineering, Faculty of Engineering, Tokyo Institute of Technology, 152-8552, Japan
*email: fakhrud.civil05@gmail.com

ABSTRACT

In a technique for widening prestressed concrete (PC) deck slabs proposed by some researchers in Japan, the shear transfer between the old and new deck slabs is achieved through the external prestressing force, and the rebars extending from the old to the new deck slab. To simulate this condition in the experimental test, three PC deck slabs under a concentrated load were tested by taking the initial prestressing level as the parameter. Observations suggest that the capacity of the widening PC deck slabs was difficult to predict due to the current analysis technique does not consider the presence of the interface between the old and new deck slab. Therefore, the conventional yield-line theory, as one of methods for calculating the flexural capacity, was modified in this study. The results indicated that the modified yield-line theory showed better accuracy compared to the conventional yield theory for lower initial prestressing level. However, for higher initial prestressing level, both conventional and modified yield line theory highly overestimated the experiment.

Keywords: Widening PC deck slab, Prestressing level, Yield-line.

1. INTRODUCTION

The new prestressed concrete (PC) box girder widening technique has been proposed by some researchers in Japan [1]. The structural members of this technique consist of the precast rib, precast PC slab, and cast-in-place new deck slab as shown in Figure 1. Full details of this technique are reported by Niwa et al. [2].

In contrast to conventional widening technique, the new technique has the potential to speed up the construction and reduce the cost because the existing PC tendons need not be extended to the new deck slab and also the use of the precast members. The shear-transfer strength between the old and new deck slabs is achieved through the external prestressing force, and through the rebars extending from the old deck slabs to the new deck slab.

Although the new widening technique has considerable merit, the failure of the widening PC deck slabs was difficult to predict due to the two-way interaction is complex and simplified analysis technique does not consider the presence of the interface between the old and new deck.

The most common failure mode for slab under concentrated load is the punching shear. Many researchers (Higashiyama and Matsui [3] ; Hamada et al. [4]; Muttoni et al. [5]; Clement et al. [6]) have proposed the equation to predict the punching shear capacities. Significant works were also performed by Mander et al. [7] to investigate the full-depth precast concrete bridge deck cantilevers, which failed in a flexural and shear mixed failure at the panel-to-panel connection. For flexure, the conventional yield line theory is one of the methods for calculating the flexural capacity of bridge deck. For this study, however, the conventional yield line theory was modified to account for the existence of the interface between the old and new deck slabs.

In this regard, this study aims to investigate the failure mechanism of the widening PC deck slabs subjected to a concentrated load. Three slabs were tested with the parameter of the initial prestressing level. The observed data were the crack patterns, deflection distributions, and strain of rebars. Finally, the experimental failure capacities were compared to the analytical predictions using the conventional and modified yield-line theory.

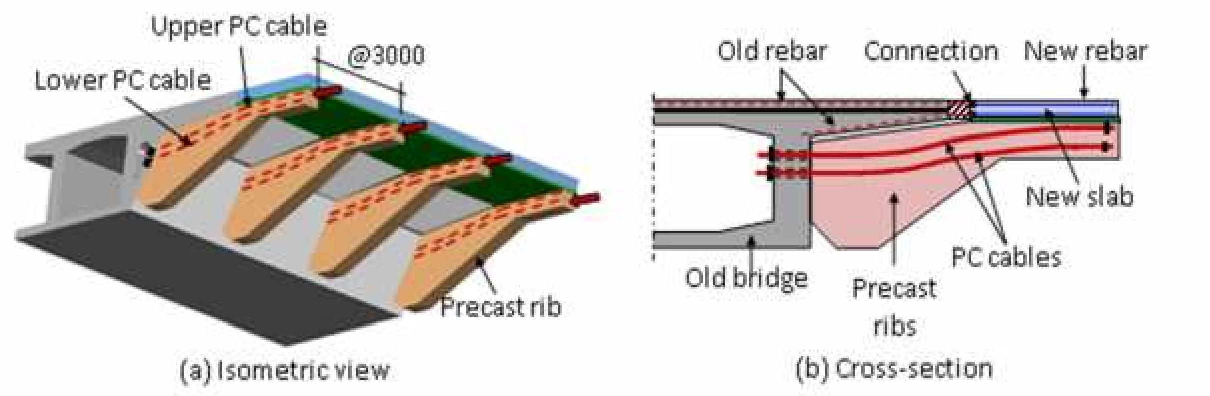


Fig 1. The new prestressed concrete (PC) box girder widening technique.

2. *Flexural Analysis For Pc Deck Slabs*

The following sections explain the analysis of flexure for normal PC deck slabs that have been cast as a single monolith slab (without interface between the old and new deck slabs). After that, this mode is subsequently to be adapted for the slabs that have interface between the old and new deck slabs.

A. Yield-line theory

Flexural failure is common in thin slabs. Sufficient shear strength is assumed so that the flexural failure mechanism governs. For such conditions, yield line theory gives an upper bound limit analysis solution for determining the collapse load of two-way slab systems based on prescribed boundary conditions. Full details of the approach are found in Park and Gamble [8]. In the yield line analysis, for a specified admissible yield line mechanism, equations of virtual work are written, unknown dimensions are determined by energy minimization, and the collapse load is calculated from Eq. 1 and Eq. 2, respectively.

$$EWD = \sum p¢_0 + \sum w_d A_d ¢_c \qquad \text{Eq. (1)}$$

$$IWD = \sum M_{px} {}^{-}{}_{x} l_x + \sum M_{py} {}^{-}{}_{y} l_y \qquad \text{Eq. (2)}$$

where P is failure load; $¢_M$ is vertical displacement below the center of the load; w_d is self-weight of the slab; A_d is area of the failure mechanism; $¢_c$ is vertical displacement at area center of the failure mechanism; M_p is the plastic moment of the slabs represented by the reinforcement crossing the yield line that can be substituted for the yielding moment along the yield lines; ¯ is rotation of the crack line and l is the length of the crack line.

B. Modified Yield-Line Theory

Inspired by Pirayeh et al. [9], the conventional yield-line theory was modified in this study which accounts for the effect of the interface between the old and new deck slabs on the failure load. Two modifications were required based on the observations from the deflection distributions and the cracking patterns. These will be explained in the next main section (Results and Discussions).

The first modification was due to the displacement distributions. For better understanding the effect of the interface, the deflection distributions of the slabs without interface and the slabs with interface are compared in Fig 2a and b, respectively. For the slabs without interface (Fig 2a), the displacement distributions from the fixed to the free support increased proportionally to the distance. On contrary, for the PC deck slab with the interface (Fig 2b), the displacement distributions from the interface to the free support tended to be constant.

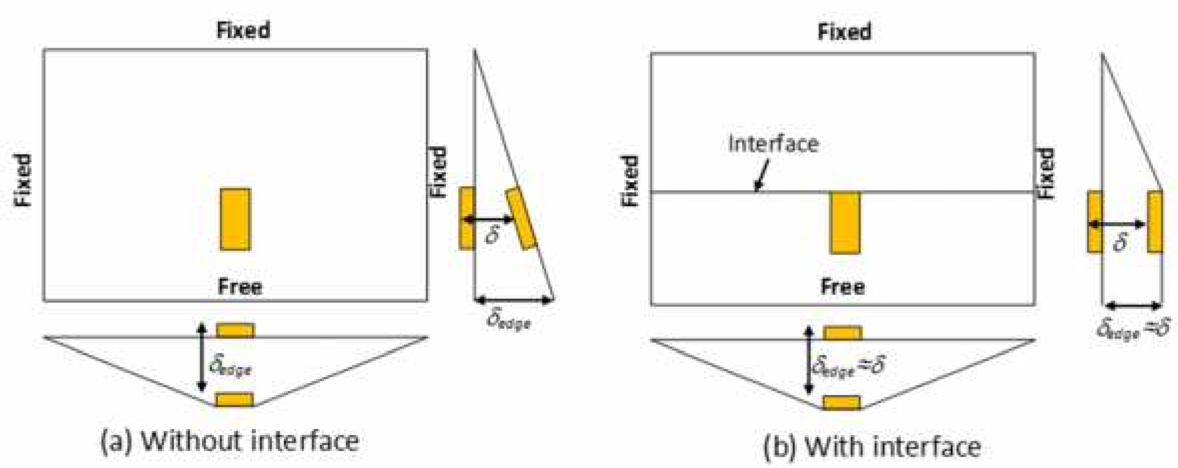


Fig 2. Influence of the interface on the deflection distributions.

The second modification was due to the additional yield line that formed at the interface between the old and new deck slabs. From the experiments, it was found that the interface between the old and new deck slabs, exactly around the loading point, was cracked at the failure. To this crack, the yield line was formed because the rebars crossing the interface (transverse rebars) have yielded at the failure.

Finally, the yield line mechanism for the PC deck slabs with the interface between the old and new deck slabs was proposed in this study as shown in Fig 3. Note that the x-direction is taken as the longitudinal direction of the deck slab axis (non-prestressed) and the y-direction is the transverse to the deck slab axis (prestressed). The yield lines are numbered from one through ten. The terms positive yield line and negative yield line are used to distinguish between those associated with tension at the bottom and tension at the top of the slab, respectively. The internal work done (EWD) remains the same as before in Eq. (1). However, the internal work done (IWD) in Eq. 2 is modified as follow:

$$IWD = 4\left(M_{x1} l_{y2} \text{¢}_o / l_x\right) + 4\left(M_{x2} l_{y1} \text{¢}_o / l_x\right) + 4\left(M_{y2} l_x \text{¢}_o / l_{y1}\right) + 2\left(M_{y2} b_1 \text{¢}_o / l_{y1}\right) \qquad \text{Eq. (3)}$$

where M_{x1} and M_{x2} are the yielding moment in the longitudinal direction of the new and old deck slabs, respectively; M_{y1} and M_{y2} are the yielding moment in the transverse direction of the new and old deck slabs, respectively; is the maximum displacement below the center of the loading plate; b_1 is width of the loading plate (100 mm in this study); l_{y1} is width of the old deck slabs (625 mm in this study); l_{y2} is width of the new deck slabs (500 mm in this study); l_x is the length of the deck slab measured from the fixed edge to the edge of loading plate (700).

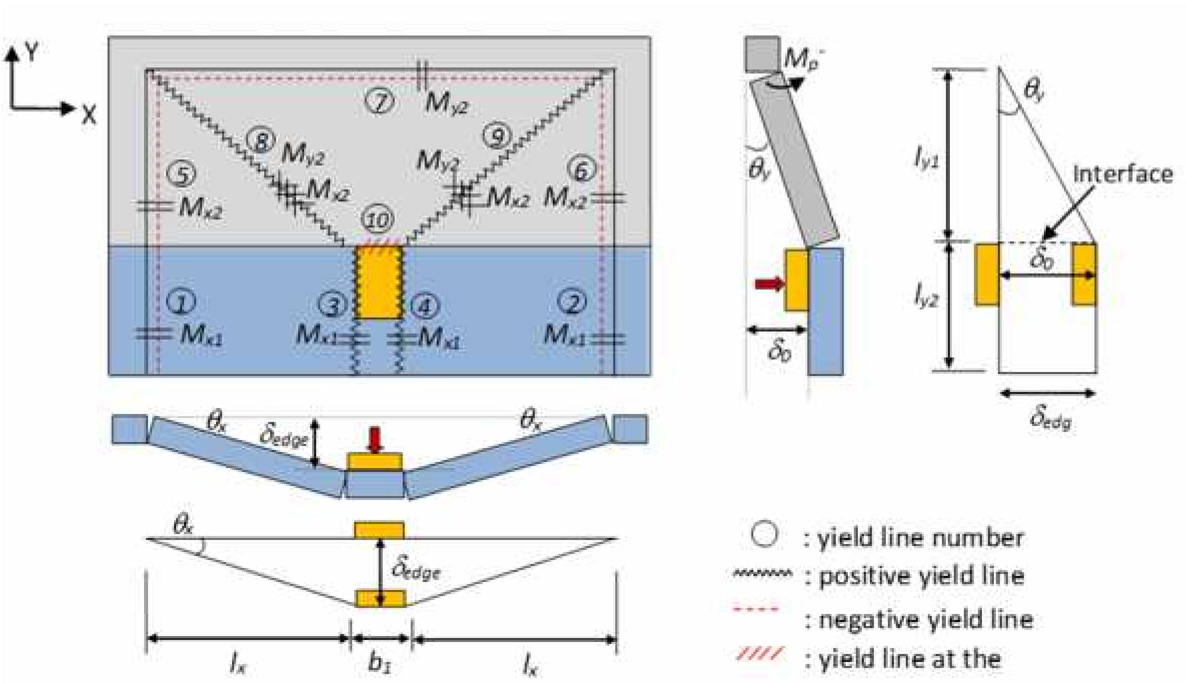


Fig 3. The assumed yield line mechanism for the PC deck slabs with interface.

3. METHODOLOGY

A. Specimen design

Three slabs were tested to investigate the effect of initial prestressing level on the load-displacement behavior, crack patterns, and identify failure modes. The initial prestressing level varied into 0.5 MPa, 1.0 MPa and 2.0 MPa in SL-P0.5, SL-P1.0, and SL-P2.0, respectively as shown in Table 1. The initial prestressing level was introduced in order to simulate the amount of the prestressing force that will be introduced to the upper PC cable in new bridge widening method.

The slabs were designed to model the slab's portion between two PC ribs having a distance of 3000 mm (Fig 1a). The half-scale model was used in this study, so that the geometrical parameters of 1500 mm long (test span), 1225 mm wide and 100 mm thick were used as shown in Fig 4. All slabs consisted of two parts and are cast at different times. The old slab is cast first followed by the new slab after seven days. The interface between the old and new deck slabs was intentionally roughened by using the retarder.

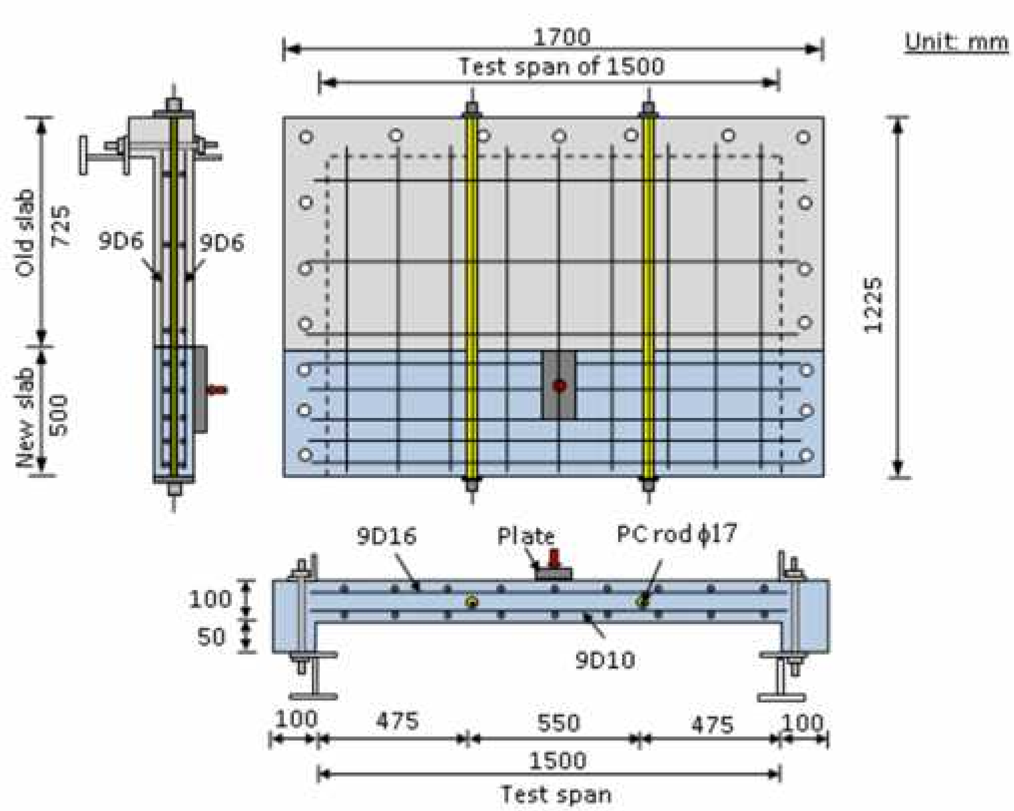


Fig 4. Deck slab layout and reinforcement details.

B. Materials

The design compressive strength of the old and new deck slabs was 50 MPa with the maximum aggregate size (G_{max}) of 10 mm. The compressive strength of concrete was determined from the compression test on 100x200 mm of cylinder specimens and tensile strength of concrete was determined from the splitting test on 100x100 mm of cylinder specimens. Both compressive and splitting tensile tests were conducted on the day of slab testing. The diameters of rebars were 6, 10, and 16 mm with the average yield strength of 345.0, 392.8, and 386.0 MPa, respectively. The yield strength f_{py}, tensile strength f_{pu}, and elastic modulus E_{ps}, of the

PC rods were 1171, 1268, and 2.01×10^5 MPa, respectively.

C. Test setup

Before testing, the slabs were prestressed using two unbonded PC rods and anchored at both ends of the slabs. After that, the slabs were restrained at the supporting steel beams and fixed with high strength steel bolts along the three edges. To achieve level surfaces, a thin layer of gypsum was applied at the interface between the supports and the slab. Finally, the slabs were tested under concentrated load by a hydraulic actuator with a maximum load of 3000 kN (Fig 5). The loading surface was 100x250 mm rectangular loading plate. This loaded area is determined from the half-scale of the footprint of the truck single-wheel load of 100 kN as specified by AASHTO LRFD 2007 [10].

During the test, the displacement and the joint opening were measured at the different points as shown in Figure 6a. The displacements were measured using six displacement transducers and the joint openings at the interface were measured using two π-gauges positioned under the loading point. Several strain gauges were also attached to the steel bars and PC rods as shown in Fig 6b.

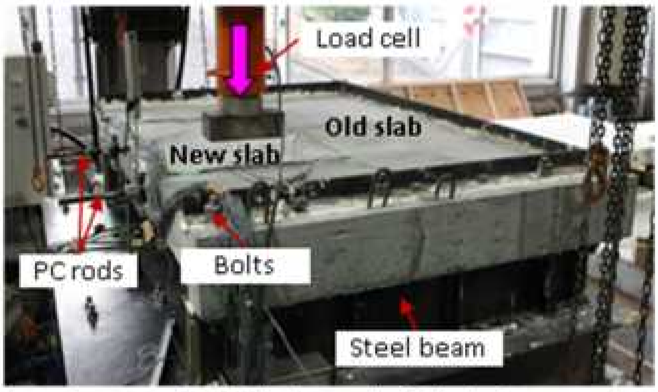
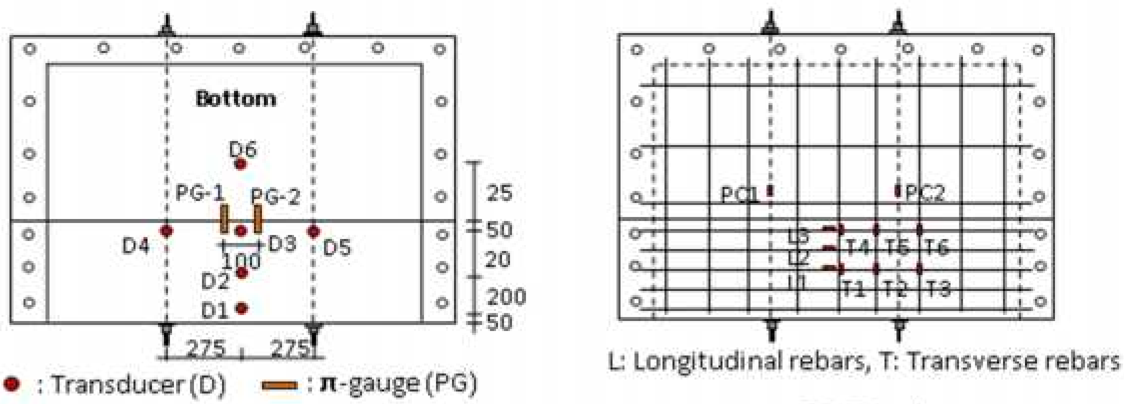


Fig 5. Photograph of the test.



(a) Location of transducers and π-gauges

(b) Location of steel gauges

Fig 6. Measurement items.

4. RESULTS AND DISCUSSIONS

A. Cracking patterns

The cracking patterns of all slabs are presented in Fig 7. The solid and dashed lines express the cracks on the top and bottom surfaces of the slabs, respectively. Safety requirements restricted access beneath the deck slab and careful mapping of the cracks. However, the cracks following the experiment are related to those of Fig 7 because the cracks were drawn after the loading test.

According to Fig 7, the failure cracking patterns was similar in all slabs which consisted of the tensile cracks on the bottom and top surface of the slabs, and shear cracks at the interface. The final cracks those appeared on the bottom surface were similar with the typical flexural yield line pattern for the slabs supported along three directions (Fig 3).

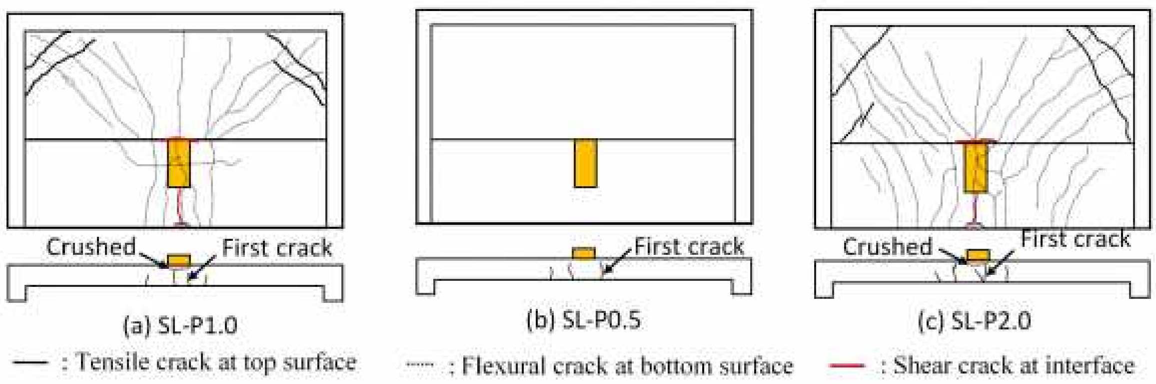


Fig 7. Experimental crack patterns.

B. Deflection distributions

The experimental results are tabulated in Table 1. The deflection distributions are presented in Figure 8. Plotted in Graphs (a) and (b) of Figure 8 are the longitudinal and transverse displacement distributions, respectively at loads of 50 kN, 75 kN, 100 kN and prior to failure. The transverse displacement distributions were plotted to show the displacement in the old and new deck slabs. This provides a useful indication on the performance of the interface, whether it adequately transfers the load to the adjacent deck.

Table 1. Experimental parameters and material properties

Specimen	σ_i (MPa)	f_c' (MPa)		f_t (MPa)		P_u *(kN)*
		Old slab	New slab	Old slab	New slab	
SL-P0.5	0.5	52.4	47.2	3.8	3.4	109
SL-P1.0	1.0	53.4	56.5	3.7	3.9	141
SL-P2.0	2.0	52.8	47.8	3.8	3.4	144

σ_i: initial prestress level at the interface between the old and new deck slabs; f_c': compressive strength of concrete and f_t: tensile strength of concrete

As shown in Figure 8a, the longitudinal displacement distributions were small at the load of 50 kN because few flexural cracks were observed at this stage. However, at the load of 100 kN, the magnitude of the longitudinal displacement was greater than the previous stage. From the recorded data, it was inferred that the first yielding of the longitudinal rebars occurred within 75 kN to 87 kN (Table 2). Prior to failure, substantial longitudinal displacement occurred because the number and width of cracks increased.

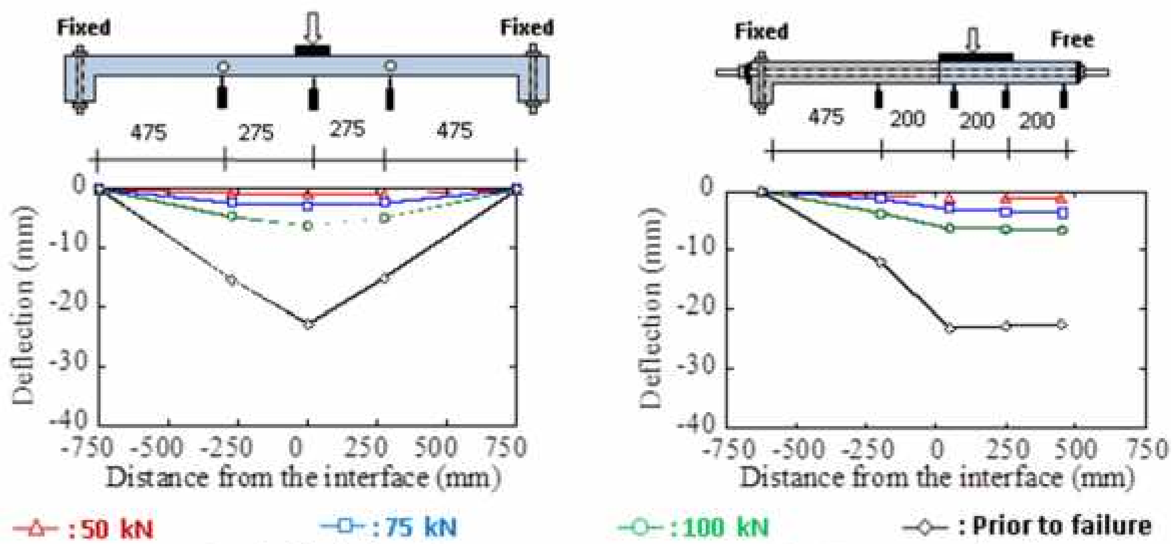
Table 2. Experimental results

Specimen	P_{cr} (kN)	P_y (kN)		P_u (kN)	J. O (mm)		□ (mm)	
		Long. rebars	Transv. rebars		JO_{cr}	JO_u	$□_{cr}$	$□_u$
SL-P0.5	53.0	75.0	71.0	109.0	0.01	1.36	1.63	16.2
SL-P1.0	63.0	87.0	103.0	141.0	0.01	2.14	1.67	22.9
SL-P2.0	77.0	82.0	114.0	144.0	0.00	1.80	2.99	32.6

P_{cr}: first joint opening load; P_y: first yielding load at rebars; P_u: ultimate load; J.O: joint opening width; □: displacement under the loading point (D3 transducer)

From the transverse displacement distributions in Fig 8b, it is evident that the interface between the old and new deck slabs remained essentially un-cracked with a smooth transition over the interface for loads up to 50 kN. In this stage, the transverse displacement distributions still behaved naturally with the cantilever structures, where the displacement increased proportionally with the distance from the fixed support to the free support. However, when the load exceeded the cracking shear stress of the interface at the load within 50 kN to 75 kN (Table 2), substantial cracking propagated at the interface, with a marked reduction in stiffness. The interface did not have a sufficient strength to transfer the shear stress to the adjacent deck slab (new deck slab).

Therefore, the transverse displacement distributions tended to be constant. This phenomenon was one of the reasons to modify the conventional yield line theory.



a. Longitudinal direction b. Transverse direction

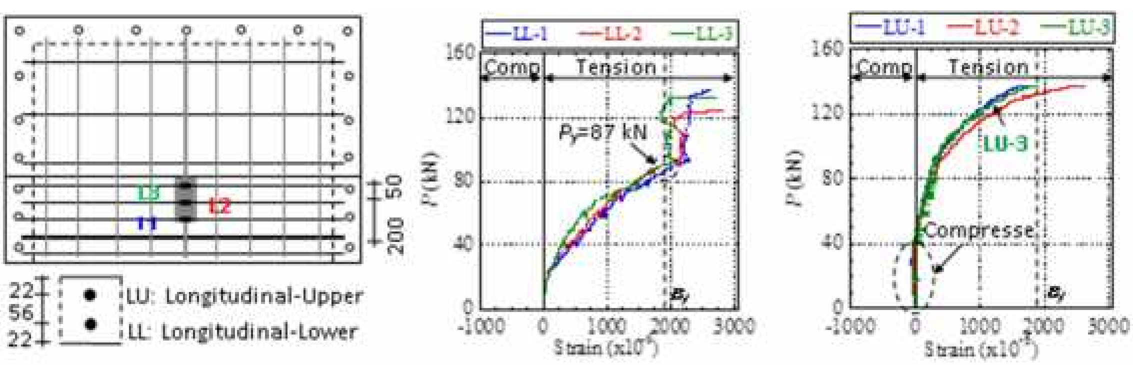
Fig 8. Deflection distributions of SL-P1.0.

C. Strain of rebars

Since the behavior of the longitudinal and transverse rebars was typical, only SL-P1.0 was described herein. Examination of the rebars strain provides a better understanding of the behavior of the interface and the failure mode of the deck slabs.

Fig 9a shows the location of the strain gauges in the longitudinal rebars. Fig 9b shows the load-strain curves in the bottom longitudinal rebars (BL). The first yield (P_y) of the bottom longitudinal rebars occurred at a load of 87 kN or 62% of the ultimate load (Table 2). At the failure, all the bottom longitudinal rebars yielded.

Fig 9c shows the load-strain curves in the top longitudinal rebars (TL). The top longitudinal rebars reached the yield load (P_y) at 132 kN or around 93% of the ultimate load (Table 2). Only TL-3 rebars which was located at the interface, near the loading point, yielded at the failure.



(a) Location of strain gauges (b) Lower layers (LL) (c) Upper layers (LU)

Fig 9. Strain in the longitudinal rebars (SL-P1.0).

Fig 10a shows the location of the strain gauges in the transverse rebars. Two strain gauges were attached at each rebars, one at 50 mm and another at 250 mm from the interface. The load-strain curves in the bottom transverse rebars (BT) are presented in Figure 10b. The yielding of transverse rebars (P_y) was initially observed in TB-6 which was located near to the interface at the load of 103 kN or around 73% of the ultimate load (Table 2). When the failure occurred, all the bottom transverse rebars at 50 mm from the interface (BT-4, BT-5 and BT-6) yielded.

Fig 10c shows the load-strain curves in the top transverse rebars (TT). The behavior of the top transverse rebars was significantly different with the bottom transverse rebars. For top transverse rebars, only the rebars located at the interface (TT-6) yielded at the failure, meanwhile the other transverse rebars were still linear elastic. TT-6 yielded because the concrete interface near the loading point failed.

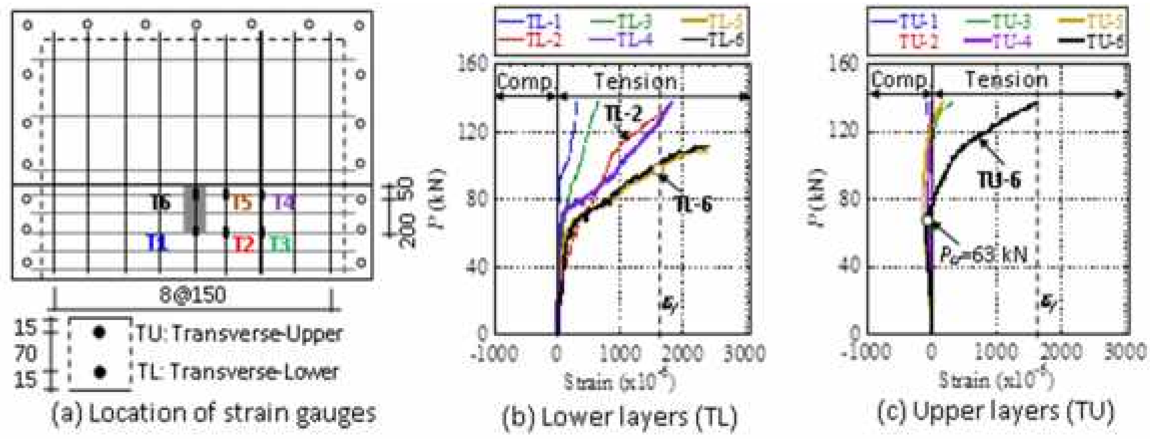


Fig 10. Strain in the transverse rebars (SL-P1.0).

5. COMPARISON EXPERIMENTAL TO CALCULATED FAILURE LOAD

The experiments are compared to the predictions of flexural capacity based on the conventional and modified yield line theory. Table 3 and 4 summarize the theoretical failure load using the conventional and modified yield line theory.

In modified yield line theory, the ultimate moment capacity of the PC deck slabs is required. It was calculated based on the maximum compressive strain along with the measured compressive strain of concrete and the yield stress of rebars. The ultimate moment capacities for all slabs are provided in Table 3. Note that the x-direction is taken as the longitudinal direction of the deck slab axis (non-prestressed) and the y-direction is the transverse to the deck slab axis (prestressed).

The comparison between the experiments to the conventional and the modified yield line theory is shown in Fig 11.

Generally, the results indicated that the modified yield line theory showed better accuracy than conventional yield line theory to predict the flexural capacity of the PC deck slabs.

Table 3. Theoretical failure load using conventional yield-line theory

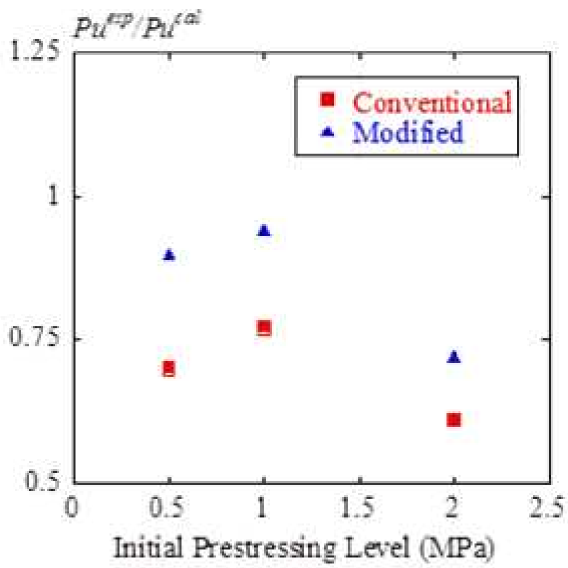
Specimen	Slab (kN-m)		EWD (kN)	IWD (kN)			$P_{CAL\text{-}CON}$ (kN)
	M_x	M_y		$\square M_x l_y \square_x$	$\square M_y l_x \square_y$	Total	
SL-P0.5	18.37	38.61	1.74	69.62	86.24	155.87	154.1
SL-P1.0	18.23	27.40	1.74	97.41	88.11	185.52	183.8
SL-P2.0	18.26	29.58	1.74	150.82	86.39	237.21	135.5

M_x: and M_y: yielding moment in the transverse and the longitudinal direction, respectively; □□: rotation of the crack line; l: length of the yield line; $P_{CAL\text{-}CON}$: the failure load calculated using the conventional yield line theory (IWD-EWD).

Table 4. Theoretical failure load using modified yield-line theory

Specimen	Old slab (kN-m)		New slab (kN-m)		EWD (kN)	IWD (kN)			$P_{CAL\text{-}MOD}$ (kN)
	M_x	M_y	M_x	M_y		$\square M_x l_y \square_x$	$\square M_y l_x \square_y$	$\square M_{int.} l_x \square_y$	
SL-P0.5	7.83	13.7	10.4	13.7	1.74	63.2	55.7	2.2	120.1
SL-P1.0	7.81	19.4	10.7	19.3	1.74	91.5	57.7	2.9	149.5
SL-P2.0	7.83	30.2	10.5	30.3	1.74	144.7	57.8	4.8	205.6

M_x: and M_y: yielding moment in the transverse and the longitudinal direction, respectively; $M_{int.}$: yielding moment in the interface; □□: rotation of the crack line; l: length of the yield line; $P_{CAL\text{-}MOD}$: failure load calculated using the modified yield line theory (IWD-EWD)



The modified yield line analysis that has been proposed herein slightly overestimated the deck slab capacity, except for SL-P2.0. For SL-P0.5 and SL-P1.0, the experimental failure loads were about 90% and 94% of the modified yield line theory, respectively. For SL-P2.0 having the highest initial prestressing level, the experimental failure load of 144 kN was only 70% of the analytical failure load of 205.6 kN from the modified yield line theory. This might be because the failure of PC deck slab was not governed by the flexural failure but other potential failure modes, such as punching shear or shear failure at the interface. Therefore, it was suggested to compare the experimental failure capacity from this study to the punching shear capacity or shear capacity of the interface obtained from the predicted equations in some existing guidelines such as JSCE [11] and *fib* Model Code 2010 [12].

For better accuracy, the applicable ranges of the modified yield-line theory was as follows: (1) the initial prestressing level: 0.5 MPa ≤ $\pm_n$ ≤ 1.0 MPa; (2) concrete strength: 33.7 MPa ≤ f_c' ≤ 69.6 MPa; (3) surface of the interface was intentionally roughened with the maximum aggregate size of 10 mm; (4) the slab is supported on three sides; and (5) PC deck slab is subjected to concentrated load near to the interface.

6. CONCLUSIONS

Based on the experimental results along with companion analyses, the following conclusions can be drawn:

1. A new calculation method for predicting the failure capacity of PC deck slab with the presence of the interface between old and new deck slabs was proposed in this study. This prediction method was proposed by modifying the conventional yield-line theory.
2. When employing the modified yield line theory for lower initial prestressing level, the analytical predictions were slightly overestimated within 4-10%, but highly overestimated around 30% of the experimental result for greater initial prestressing level.

7. ACKNOWLEDGEMENT

The authors are would like to grateful to Fuji P.S. Corporation for their kind support to this research project.

8. REFERENCES

[1] Yamauchi, T., Morita, A., Sato, A., Nishinaga, T., Tokumitsu, T., Yagi, Y., Yamada, M. and Yamaguchi, M., *Widening PC floor version structure and widening construction method of established PC floor version*. Japanese unexamined patent application publication No. 2016-61045, 2016-04-25, 2016.

[2] Niwa, J., Fakhruddin, Matsumoto, K., Sato, Y., Yamada, Y. and Yamauchi, T., "Experimental study on shear behavior of the interface between old and new deck slabs", *Engineering Structures*, Vol. 126, pp. 278-291, 2016.

[3] Higashiyama, H. and Matsui, S., (1998). "Fatigue durability of longitudinally prestressed concrete slabs under running wheel." *JSCE J. Struct Mech Earthq Eng*, 605/I-45, 79-90. (in Japanese)

[4] Hamada, S., Yang, Q. and Mao, M., (2008). "Evaluation of punching shear strength of reinforced concrete slabs based on data base." *Journal of Advanced Concrete Technology*, 6(1), 205-214.

[5] Muttoni, A. and Ruiz, M., (2012). "The levels-of-approximation approach in MC2010: Application to

punching shear provisions." *Structural Concrete*, 13(1).

[6] Clement, T., Ramos, A., Ruiz, M. and Muttoni A., (2013). "Design for punching of prestressed concrete slabs." *Structural Concrete*, 12(2), 157-167.

[7] Mander, T. J., Mander, J. B. and Hite Head, M., (2011). "Modified yield line theory for full-depth precast concrete." *J. Bridge Eng.*, 16(1), 12-20.

[8] Park, R., and Gamble, W. L., *Reinforced concrete slabs*, 2nd Edition, Wiley, New York, 2000.

[9] Pirayeh Gar, S., Mander, J. B., Head, M. and Hurlebaus, S.: FRP slab capacity using yield line theory, *J. Compos. Constr.*, Vol. 18 No. 6, pp. 1-10, 2014.

[10] AASHTO LRFD, (2007). "Bridge design specifications." *American Association of State Highway and Transportation Officials*.

[11] Japan Society of Civil Engineers, "Standard specification for design of concrete structures, structure performance verification", 2007.

[12] International Federation for Structural Concrete (*fib*): Model code 2010-final draft, Vol. 1, March 2012.